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Continuous Measurement Enhanced Self-Trapping of Degenerate Ultra-Cold Atoms in a Double-Well: Nonlinear Quantum Zeno Effect

WeiDong Li[1,2], Jie Liu[1*]
[1] *Institute of Applied Physics and Computational Mathematics, Beijing, 100088, People's Republic of China and*
[2] *Department of Physics and Institute of Theoretical Physics, Shanxi University, Taiyuan 030006, China*

In the present paper we investigate the influence of measurements on the quantum dynamics of degenerate Bose atoms gases in a symmetric double-well. We show that continuous measurements enhance asymmetry on the density distribution of the atoms and broaden the parameter regime for self-trapping. We term this phenomenon as nonlinear quantum Zeno effect in analog to the celebrated Zeno effect in a linear quantum system. Under discontinuous measurements, the self-trapping due to the atomic interaction in the degenerate bosons is shown to be destroyed completely. Underlying physics is revealed and possible experimental realization is discussed.

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I. INTRODUCTION

Double-well system is a paradigm to demonstrate quantum tunnelling phenomena and attracts much attention from the societies of both theoreticians and experimentalists since the establishment of quantum mechanics [1]. For a symmetric double-well, the amplitude distributions of all eigenstates are symmetric, so a particle is expected to oscillate between the two wells and its chance to visit each well is even [2]. However, there exist two effects that can break the above symmetry leading to an asymmetry on the particle's density distribution, i.e., quantum measurements [3, 4, 5, 6, 7, 8, 9, 10] and collisions between particles (many-body effect) [11, 12, 13, 14, 15]. In the former case, the quantum measurement couples the system to outer environment and induces the quantum de-coherence of the system. As a result, the quantum tunnelling between two wells is suppressed completely so that the particle keeps staying in one well and has no chance to visit the other well. This is the celebrated quantum Zeno effect (QZE) [9, 10]. In the latter case, the two-body collisions among the degenerate Bose-Einstein condensates (BECs) atoms leads to a nonlinear excitation, manifesting a highly asymmetric density profile of the BECs even in a symmetric double-well. This somehow counterintuitive phenomenon has been observed recently in labs [13, 14, 15].

In the present paper, what we concern is how a measurement affects the dynamics of the many-body quantum system characterized by degeneracy and diluteness. Under a mean-field approximation and without measurement, the dynamics of this system is described by a nonlinear Schrödinger equation, known as Gross-Pitaevskii equation (GPE), where the nonlinearity arises from the interaction between the degenerated atoms and is responsible for the unusual self-trapping phenomenon. Considering the measurement carried out by a position meter, the GPE is modified to be a stochastic nonlinear differential equation. With solving the stochastic equation we achieve insight into how the measurement affects the self-trapping of BECs. Our main result is that continuous position measurements enhance asymmetry on the distribution of the atoms density profile and broaden the parameter regime for self-trapping. We term this phenomenon as nonlinear quantum Zeno effect. However under discontinuous measurements, the self-trapping phenomenon could be destroyed. Physics behind the above phenomena is revealed and possible experimental realization is discussed.

This paper is organized as follows: We first briefly introduce the quantum measurement theory in Sec. II. In the third section, we describe our quasi 1-D double-well model of BECs. In Sec.IV, we discuss the influences of the continuous or discontinuous measurements on the dynamics of the system. Finally, we summarize and discuss our work in section V.

II. MEASUREMENT THEORY

In the physics community, the measurement side of quantum theory is one of the fundamental issues [16, 17, 18]. There exist various theories about this topic depending on concrete physical systems and measured physical quantities [19, 20]. For the BECs system we consider, we make use of continuous (in time) position measurement theory [9, 10]. Within the framework of this theory, the position and the momentum of the system and the meter are denoted by $\{x(t), p(t)\}$ and $\{X(t), P(t)\}$, respectively. Usually we can describe the pseudo-classical meter as $\langle Q|X(t), P(t)\rangle = \left(2\pi\sigma^2\right)^{-1/2} \exp\left\{-\frac{[Q-X(t)]^2}{4\sigma^2}\right\} \exp\left\{\frac{i}{\hbar} QP(t)\right\}$ in the Q (position of the meter) representation, where σ denotes the uncertainty of the meter pointer. Considering that the meter is pseudo-classical, one can treat this state as $|X(t), 0\rangle$

$$|X(t), 0\rangle = \left(2\pi\sigma^2\right)^{-1/2} \exp\left\{-\frac{[Q-X(t)]^2}{4\sigma^2}\right\}. \quad (1)$$

This is one of the features of this theory that the higher up the chain from the system to the observer the cut is placed, the more accurate the model of the measurement will be. The interaction between the system and the meter can be described as

$$\hat{H}_{\mathrm{SM}} = \gamma \hat{P} \left[\hat{x} - \langle x \rangle_s \right], \tag{2}$$

where $\langle x \rangle_s$ is the average position of the system and γ represents the relaxation rate of the meter pointer to the system mean position $\langle x \rangle_s$. This interaction translates the position of the meter by an amount proportional to the average position of the system. The state of the system-meter at time t is taken to be

$$|\Phi(t)\rangle = |X(t), 0\rangle \otimes |\Psi(t)\rangle,$$

where $|\Psi(t)\rangle$ is a system state vector. The combined state at time $t+\tau$ is

$$|\Phi(t+\tau)\rangle = \exp\left\{ -\frac{i\gamma\tau}{\hbar} \hat{P} \left[\hat{x} - \langle x \rangle_s \right] \right\} |X(t), 0\rangle \otimes |\Psi_0(t+\tau)\rangle, \tag{3}$$

where

$$|\Psi_0(t+\tau)\rangle = exp\left(-\frac{i\gamma\tau}{\hbar} \right) |\Psi_0(t)\rangle. \tag{4}$$

From Eq.(3) and Eq.(4), we can see that the evolution of the combined system-meter is purely unitary. The meter has undergone the desired translation because of the interaction with the system. Meanwhile the system has undergone its usual free evolution combined with a measurement back-action from the meter pointer.

After considering the read-out, the evolution of the combined equations in the first order of τ can be written as a stochastic differential equation

$$\begin{aligned} \frac{d}{dt}|\Psi(t)\rangle \; &= \; (-\frac{i}{\hbar}H_0 - \frac{\Gamma}{2}\left[x - \langle \hat{x} \rangle_s\right]^2 \\ &\quad + \sqrt{\Gamma}\xi(t)\left[x - \langle \hat{x} \rangle_s\right])|\Psi(t)\rangle \end{aligned} \tag{5}$$

where $\Gamma = \gamma^2\tau/8\sigma^2$ denotes the interaction strength between the meter and the system. The noise term $\xi(t)$ indicates Gaussian noise of standard deviation 1 and can be modelled by white noise $\langle \xi(t)\xi(t') \rangle = \delta(t-t')$.

The above stochastic Schrödinger equation is equivalent to a stochastic master equation for the selective evolution of the conditioned density operator [9]. It is conditioned on the entire history of the meter readout X(t). If we were only interested in the non-selective evolution of the system, then we would have to discard all knowledge of the evolution of the system. This is achieved in the usual manner of averaging over all possible meter readouts at all times t. In our case this simply amounts to averaging over the stochastic term in Eq.(5), which gives zero. The process of photon scattering on a BEC that results in population difference measurement can be modelled by the non-selective evolution of the density operator [21]. In our following discussions, for generality, we include both the de-coherence term and the noise term modelling the nondestructive measurements on BECs.

III. DOUBLE-WELL MODEL

We will focus on the case of BECs trapped in a quasi-one dimensional symmetrical double-well. Along the lateral directions the BECs is tightly confined so that the Hamiltonian governing the dynamics of BECs reduces to the following 1D form,

$$\hat{H}_0 = -\frac{1}{2}\frac{\partial^2}{\partial x^2} + V(x) + \eta|\psi(x)|^2, \tag{6}$$

after re-scaling the energy unit by $\frac{\hbar^2}{mL^2}$ and the length unit by L (effective size of the double-well potential); m is the mass of the trapped atom and η is the nonlinear parameter that is proportional to the 1-D reduced s-wave scattering length between the degenerate atoms and the total number of the atoms. As the size of the two well potential realized in the experiment [13] is around $L \sim 13\mu m$ and the mass of the alkali atom is around $m \sim 10^{-25}kg$, the order of the time in our paper should be

$$\frac{mL^2}{\hbar} \sim \frac{10^{-25} \times 10^{-12}}{10^{-34}} \sim 10^{-3}s.$$

200 time duration in our dimensionless unit corresponds to $0.2s$, that is within the lifetime of the BECs under the present experimental conditions. So, our calculations are extended up to time moment of 200 in following. Meanwhile, the energy scale is around khz. The double-well potential is expressed by

$$V(x) = -\frac{1}{4}x^2 + \frac{1}{64}x^4. \tag{7}$$

To investigate the tunnelling dynamics of BECs in this double-well, we start from a superposition of ground and the first excited state of stationary GPE, i.e., $\hat{H}_0\psi_{0,1}(x) = \mu_{0,1}\psi_{0,1}(x)$, where $\psi_{0,1}(x)$ are ground state and the first excited wave function respectively and the corresponding chemical potential are $\mu_{0,1}$. The coefficients of the superposition are chosen so that the BECs are localized in one well initially, i.e., $t = 0$, as shown in Fig.1.

$$|\Psi(x,0)\rangle = \frac{1}{\sqrt{2}}\left(\psi_0(x) \pm \psi_1(x)\right). \tag{8}$$

We should mention that, in addition to the above symmetric states having linear counterpart, the GPE allows eigenstates that are totally asymmetric, so called symmetry-breaking states, in the strong interaction case. These asymmetric eigenstates are crucial in understanding the dynamics of BECs and are the source of unusual self-trapping phenomenon [22, 23]. Our initial state is chosen as a superposition of ground and the first excited states and has asymmetric property, but it has nothing to do with the above asymmetric eigenstates. And further, because the superposition principle breaks down in the nonlinear case [23], we cannot predict the temporal evolution of a superposition state from the superposition principle.

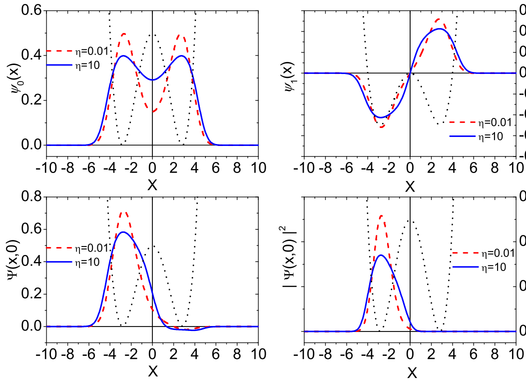

FIG. 1: (Color online) The top two pictures show ground state and the first excited state, respectively. The bottom two show the wave function and the density profile of the initial sate, respectively.

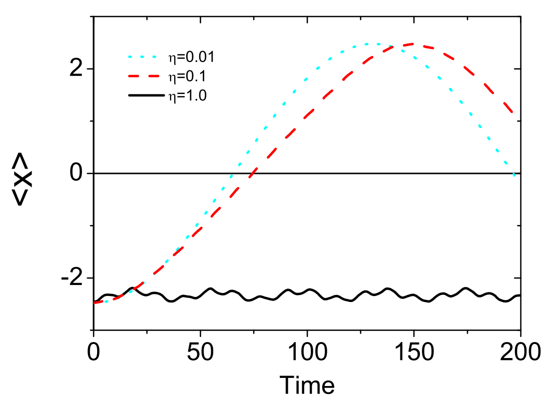

FIG. 2: (Color online) The Josephson Oscillation and Self-trapping effect at $\Gamma = 0$

In the absence of the nonlinear interaction between atoms, i.e., $\eta = 0$, our Hamiltonian (6) reduces to one depicting the motion of single particle in double-well potential and the QZE is observed in the presence of continuous measurements [9, 10]. When the nonlinear interaction is present and larger than a critical value η_c [2, 11, 12], the somehow counterintuitive self-trapping phenomenon occurs even without performing measurement. In the following part of this section, the above interesting behavior will be demonstrated. Under the combined nonlinear interaction and quantum measurements, the dynamics of BECs is dramatically influenced and detailed discussions will be presented in next section.

Without performing measurements, i.e., $\Gamma = 0$, the dynamics of the system strongly relies on the interaction between the particles, the time-dependent GPE governs the evolution of the BECs. As in Ref. [12], we apply the operator-splitting method to calculate the evolution of the wave function and then calculate average position through the formula $\langle x(t)\rangle = \int \psi(x,t)^* x \psi(x,t) dx$. In Fig.2, we show the dynamic evolution of the coherent superposition state (8) for different nonlinear interaction parameters η by plotting the average position $\langle x(t)\rangle$ with respect to time. It shows quite different dynamic behaviors, depending on the values of the nonlinear interaction. For the weak interaction case, i.e., $\eta < \eta_c \sim 0.143$, the BECs demonstrate a coherent Josephson oscillation between two wells, while for $\eta > \eta_c$, the BECs are trapped in one well, a phenomenon so called as the self-trapping. In our case, the population difference between the two wells of our initial state is quite large, so the self-trapping condition is easier to be satisfied compared to usual cases [2, 11].

IV. THE INFLUENCE OF THE MEASUREMENT

In this section, we will investigate the influence of the measurements on the dynamics of the system. According to the measurement theory, the continuous measurements is modelled by treating a sequence of n measurement operations described in the Sec.I, each separated by a time interval Δt, over a total time duration $T = n\Delta t$. The continuous limit is obtained by taking the limit $\Delta t \to 0$ [9]. The probability that the system is found on the initial state at each independent measurement is $P_0(t = T) \approx 1 - 2(\Delta H)^2 \frac{T^2}{n}$, where $(\Delta H)^2 = \langle\psi_0|H^2|\psi_0\rangle - \langle\psi_0|H|\psi_0\rangle^2$. As is shown in following, one can recover the QZE in the linear dynamics of the system ($\eta = 0$ in (6)) by setting the time step as $\Delta t = 10^{-3}$, which corresponds to the separated time τ between two consecutive measurements. But if we increase the time interval to $\Delta t = 10^{-2}$, the continuous measurements condition is broken, we fall into the regime of the discontinuous measurements.

A. Continuous measurements: The nonlinear Zeno effect

With applying the continuous measurements, the dynamics of the system is governed by Eqs.(5). It is interesting to note that the equation (5) can be split into two parts. One is the normal free evolution part of the nonlinear Schrödinger equation, the other part is the time-dependent random part. To solve the dynamic evolution of this problem, we first transform the wave function into the "interaction" representation and integrate the time-dependent part (the second part) with the Weiner increment method. Then we apply the inverse operation to obtain the wave-function in the coordinate space. Please note that the transformation to the "interaction" representation is time-independent unitary transformation described as $\exp(-iH_0\Delta t/\hbar)$, where Δt is the time step.

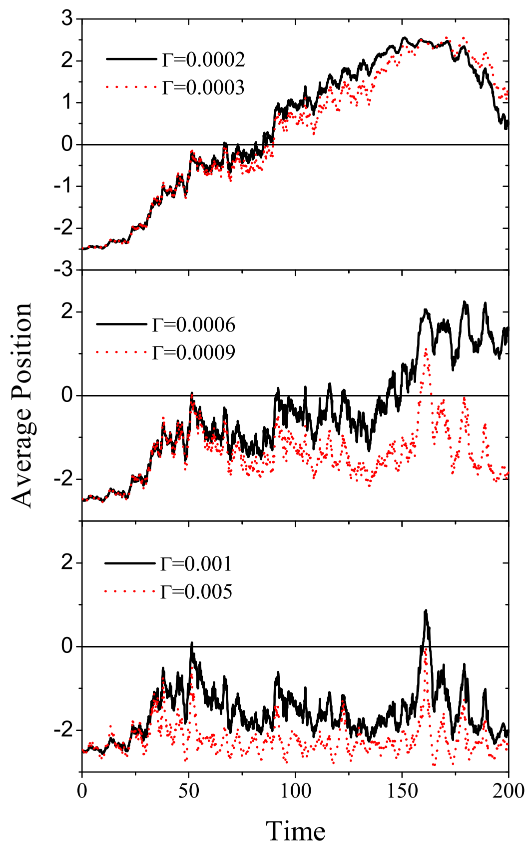

FIG. 3: (Color online) The average position of the BECs as the function of time with increase the value of Γ from the top to the bottom in the linear case.

The Weiner increment dw helps us to numerically integrate Eqs.(5) with a random term. According to the theory of Stochastic different equations, we introduce dw as follows,

$$\frac{dw}{dt} = \xi\left(t\right). \tag{9}$$

Considering that our noise term $\xi(t)$ is Gaussian white noise [24], we can write Weiner increment as

$$dw = \sqrt{\tau}\ \mathrm{Rand}\left(0, 1\right), \tag{10}$$

where $\mathrm{Rand}(0, 1)$ denotes the Gaussian random number with zero mean value and 1 standard variance.

Besides the time-dependent random noise term, we also have to include the idea of the quantum trajectory theory [9, 10]. In the quantum trajectory theory, each measurement operator can be regarded as a "state preparation" procedure. This procedure creates an ensemble of systems each possessing a certain state. Actually, the measurement operator is realized by projecting the state into one of the position meter state (1). Physically, we can understand this process as following: After finishing each measurement, we have destroyed the measured quantum state and the state collapses into one of the states of meter pointer. So in the next step, we exactly start our evolution from this quantum state. Therefore, after each measurement quantum state is prepared through a normalization process.

In Fig.3, we plot the temporal evolution of average position of the particle for linear case $\eta = 0$. It shows that, for weak measurement (i.e., small Γ), the particle oscillates between the two wells, whereas the particle will stay in the left well (its initial location) with increasing the interaction strength between the meter and the system. The above calculation clearly demonstrates the celebrated QZE. In the above calculation, the time step is set as $\Delta t = 10^{-3}$, which is small enough to guarantee that our measurements are continuous. Without measurement, the period of the oscillation is around $T_{os} \sim 250$. From our Eq.(5), the de-coherence term is estimated proportional to $\Gamma/2 \times (\Delta x_0)^2$, where Δx_0 is the distance between two wells and is $2\sqrt{8}$. So it is expected that when $\Gamma \sim (2 \times \pi)/(16 \times T_{os}) \sim 0.0015$ the de-coherence effect caused by the measurement will be significant so that the linear QZE is observed. The above simple estimation excellently agrees with our numerical calculation as shown in Fig.3, where we see at $\Gamma > 0.001$ the particle turns to be localized in its initial well.

However, we find that the above simple picture is no longer available when the nonlinear interaction between atoms emerges. In this situation, we find that the decoherence effect caused by the measurement is significantly enhanced by the atomic interaction.

We plot the average position of BECs in the presence of the nonlinear interaction between atoms, i.e., $\eta = 0.1$, in Fig.4. In this case, the nonlinear interaction is smaller than the critical nonlinear interaction η_c, so that the BECs show a Josephson oscillation between the two wells in the absent of measurements $\Gamma = 0$ (see Fig.2). With increasing the interaction strength between system and meter, Fig.4 shows that the BECs turn to stay in one well manifesting that the averaged position is smaller than zero. The critical value Γ for the occurrence of Zeno effect is modified from $\Gamma = 0.001$ in linear case to $\Gamma = 0.0003$ in weak nonlinear case ($\eta = 0.1$). In this case the period of the nonlinear Josephson oscillation is $T_{os} \sim 300$. From the simple picture given in the above discussions we estimate that the de-coherence effect become significant only when $\Gamma > 0.0012$. However, at $\Gamma = 0.0003$ we already have observed the blockage of the Josephson oscillation. This analysis implies that the interaction among BECs atoms significantly enhance the de-coherence effect. As a result, even a moderately weak nonlinear interaction can play a dramatic role and significantly change the movement of the atoms. So we conclude that the nonlinear interaction broadens the regime of the localization of atoms and makes it easy to observe the quantum Zeno effect. Comparing with the linear quantum Zeno effect, we term this phenomenon as nonlinear quantum Zeno effect. On the other hand, the continuous measurements suppress

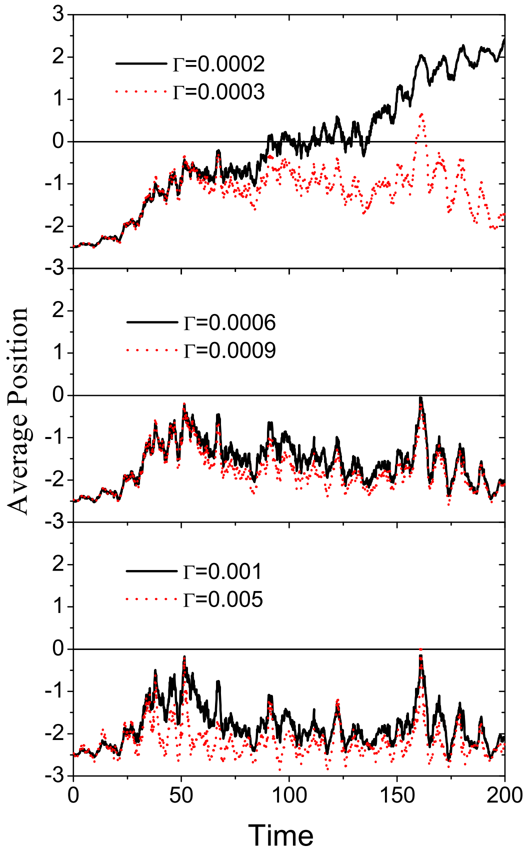

FIG. 4: (Color online) Same as Fig.(3) in the case of $\eta = 0.1$

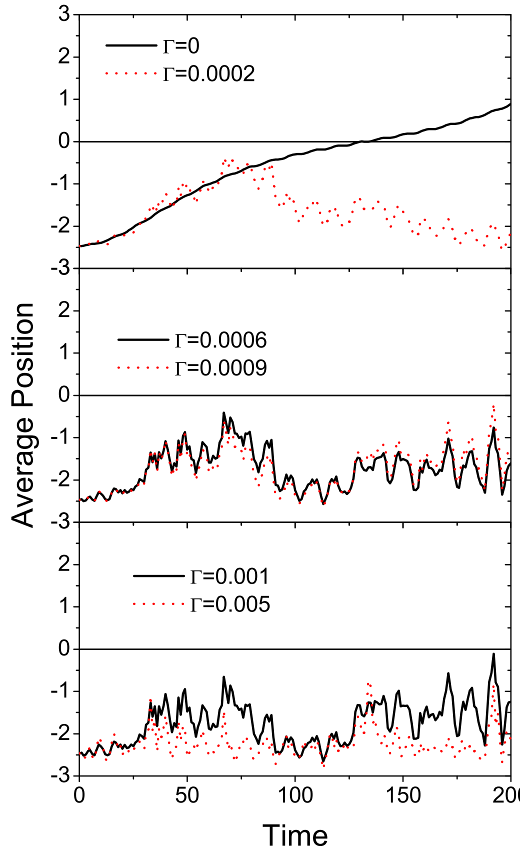

FIG. 5: (Color online) Same as Fig.3 in the case of $\eta_c = 0.143$

the oscillation between the two wells and enhance the self-trapping by enlarging the regime of self-trapping. To show this point more clearly, we further plot the average position of the BECs with $\eta_c = 0.143$ in Fig.5.

When the nonlinear interaction is larger than the critical value, we also calculate the average position of BECs with respect to time as shown in Fig.6. In the presence of the strong nonlinear interaction, the BECs atoms are self-trapped even without performing the measurement. From Fig.6, we see that, the continuous measurements only cause small displace of the averages position of BECs, but can not drive the BECs to tunnel through the barrier. This means that the continuous measurements do not destroy this kind of nonlinear self-trapping.

In Fig.7, we plot the parameters diagram of the occurrence of the self-trapping. We find that, the continuous measurements broaden the parameter regime for the emergence of self-trapping. For example, the continuous measurements of strength $\Gamma = 0.0006$ reduces the value of the critical nonlinear interaction η_c from 0.143 to 0.05. On the other hand, the nonlinear interaction between the degenerate atoms makes it easier to observe QZE by reducing the critical coupling strength Γ from 0.001 in linear case (see Fig.3) to 0.0003 in the case of $\eta = 0.1$ (see Fig.4). Therefore, we conclude that, the combination of the continuous measurements and the nonlinear interaction greatly broaden the parameter regime for observing QZE and occurrence of the self-trapping.

B. discontinuous measurement

So far, we have investigated the effect of the continuous measurements on dynamics of the nonlinear quantum system. In this part, we want to know what happen if our measurement is not continuous, i.e., to study the influence of the discontinuous measurements on the self-trapping effect.

To investigate the effect of discontinuous measurements, we will modify the time interval of the sequence of the measurement from 10^{-3} to 10^{-2}. Considering the barrier height, we can not increase the parameter Γ to too large. But if we keep $\Gamma = 0$ and increase the nonlinear interaction to $\eta = 0.5$, the dynamics of BECs initially localized in one well demonstrates a self-trapping phenomenon the same that as shown in Fig.2. In Fig.8, we plot our results in the same way as the previous figures from the top

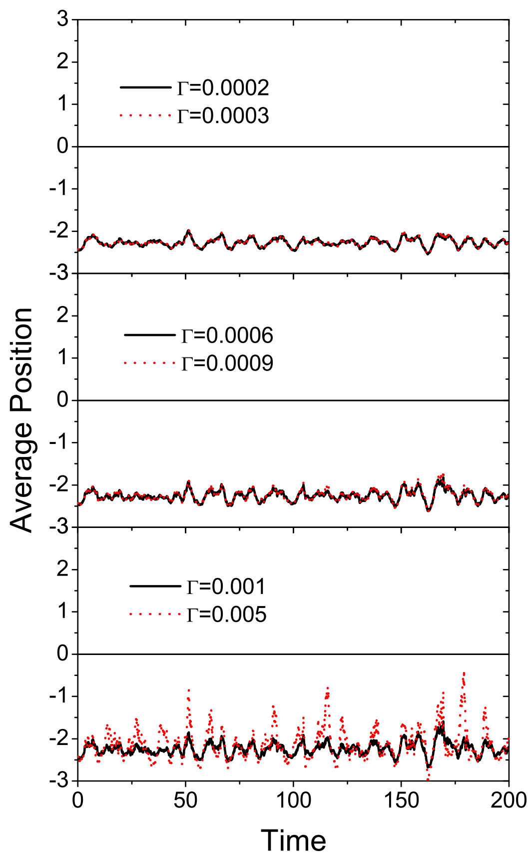

FIG. 6: (Color online) Same as Fig.3 in the case of $\eta = 1.0$

to the bottom for various values of Γ. It shows that the oscillation amplitude of the averaged position of BECs increases with the increasing measurement strength Γ. In the case of $\Gamma = 0.005$, the particles can cross the barrier and tunnel into the other well. This means that the discontinuous measurements break the self-trapping effect that is induced by the strong interaction between atoms. In the other words, the discontinuous measurements but still high frequency measurements have tendency to recover the macroscopic coherence properties of the system. The similar effect of the measurements process has been pointed out by investigating the dynamics of the quantum dynamics of the BECs with two wells including the nondestructive measurements [25].

V. CONCLUSION

In conclusions, we have investigated the influence of both continuous and discontinuous position measurements on the quantum dynamics of degenerate Bose atoms gases in a symmetric double-well. We find that, continuous position measurements enhance the asymmetric distribution of the atoms density profile and broaden

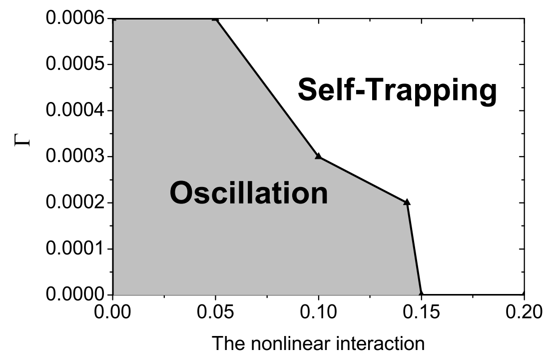

FIG. 7: The diagram of the parameter regime of the self-trapping, the light grey region shows the regime of the coherent oscillation

the parameter regime for self-trapping. Moreover, the de-coherence effect caused by the measurement is significantly enhanced by the atomic interaction. The nonlinear interaction between the degenerate atoms makes it easier to observe QZE by decreasing the critical coupling strength Γ, only over which observing QZE is possible. Therefore, we conclude that, the combination of the continuous measurements and the nonlinear interaction greatly broaden the parameter regime for observing QZE and occurrence of the self-trapping. On the other hand, we find that, discontinuous measurements may break the self-trapping. It implies that the discontinuous position measurements may enhance the tunnelling of the BECs atoms. In the present experimental condition, BECs trapped in the symmetric double-well has been realized with using optical trap technique [13] and nondestructive measurements can be realized by shining a coherent light beam through BECs [25]. The measurement of the mean coordinate is, roughly speaking, equivalent to that of the inter-well population difference. Therefore, the photon scattering on BECs in a double-well potential is suggested to give the population difference measurement [21]. Our predicted phenomena is hopefully observed with present experimental technique.

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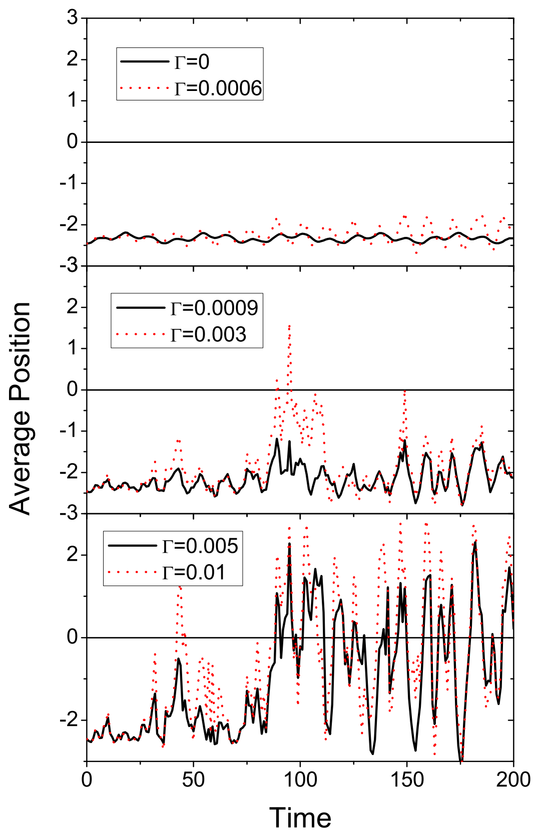

FIG. 8: (Color online) The average position as the function of time for time step $\Delta\tau = 10^{-2}$ with the nonlinear interaction is $\eta = 0.5$

[*] Liu_Jie@iapcm.ac.cn

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